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Perfect space and time focusing ion optics for multiturn time of flight mass spectrometers

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Abstract

Ion optics of multiturn time-of-flight mass spectrometer that satisfy both perfect space and time focusing were investigated. The basic concepts of perfect focusing are explained. It was found that the perfect focusing conditions can be reduced by introducing symmetrical geometries. The conditions are expressed in the transfer matrix form. (Int J Mass Spectrom 197 (2000) 179–189) © 2000 Elsevier Science B.V.

Keywords: Time-of-flight mass spectrometer; Multiturn; Perfect focusing; Ion optics

1. Introduction

Since time-of-flight mass spectrometry (TOF/MS) was introduced in the 1950s, TOF/MS has grown as a useful mass analytical method in various fields. The TOF mass spectrometer has become an especially powerful and attractive instrument after the introduction of laser ionization and very fast electronics.

A major limitation of TOF/MS has been its relatively poor mass resolution. In order to improve the mass resolution, the following three types of methods were successfully introduced. (1) In order to minimize the effects of spacial and velocity dispersion on time resolution, two-stage acceleration, time-lag focusing [1], and orthogonal acceleration [2] methods were developed. (2) In order to satisfy energy focusing, electric sectors [3,4] or electric mirrors [5] have been used. (3) In order to employ a long flight length, multiturn ion optical geometries were introduced [3,6], but they did not satisfy the perfect focusing condition. Therefore ion beam divergence and mass resolution decrease with an increasing number of turns around the system. To avoid this problem, the ion optical systems should satisfy the perfect focusing condition.

We have already found the ion optical systems, which satisfy the perfect focusing condition and one of these proposed TOF systems have been constructed [7]. In this article, we discuss the general principle of the perfect space and time focusing conditions for a multiturn TOF mass spectrometer.

2. Principle of perfect space and time focusing

2.1. Transfer matrix method to express the ion trajectories

Generally, there are two methods to calculate the ion trajectories in electromagnetic fields, namely: (1) the transfer matrix method and (2) the ray tracing

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method. In this article the transfer matrix method is used and the definition of notation is described in our previous article [4].

The geometrical trajectory of an arbitrary particle can be expressed by an ion optical position vector (x, α, y, β, γ, δ). Here x, y, and α, β denote the lateral and angular deviations of the ion under consideration from a reference ion at the object. The mass and energy deviations, γ and δ, are defined as:

$$m/e = (m_0/e_0)(1 + \gamma), \qquad U/e = (U_0/e_0)(1 + \delta) \tag{1}$$

where m, U, and e are, respectively, the mass, energy, and charge of an arbitrary ion and m_0, U_0, e_0 are those of a reference ion. In order to describe flight time, the concept of the path length deviation l is added to the original ion optical position vector. The new position vector (x, α, y, β, γ, δ, l) at an arbitrary profile plane can be related to the initial position vector (x_0, α_0, y_0, β_0, γ, δ, l_0) in a first order approximation by a transfer matrix A in the following way:

$$\begin{pmatrix} x \\ \alpha \\ y \\ \beta \\ \gamma \\ \delta \\ l \end{pmatrix} = \begin{pmatrix} A(x|x) & A(x|\alpha) & 0 & 0 & A(x|\gamma) & A(x|\delta) & 0 \\ A(\alpha|x) & A(\alpha|\alpha) & 0 & 0 & A(\alpha|\gamma) & A(\alpha|\delta) & 0 \\ 0 & 0 & A(y|y) & A(y|\beta) & 0 & 0 & 0 \\ 0 & 0 & A(\beta|y) & A(\beta|\beta) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ A(l|x) & A(l|\alpha) & 0 & 0 & A(l|\gamma) & A(l|\delta) & 1 \end{pmatrix} \begin{pmatrix} x_0 \\ \alpha_0 \\ y_0 \\ \beta_0 \\ \gamma \\ \delta \\ l_0 \end{pmatrix} \tag{2}$$

If the system consists of several ion optical components such as electric sectors, quadrupole lenses, drift spaces, then the overall transfer matrix R can be simply obtained by multiplying only the transfer matrix of the individual elements as

$$R = A_n \times A_{n-1} \times \cdots \times A_2 \times A_1 \tag{3}$$

Focusing conditions of a whole system will be discussed using the above overall transfer matrix elements $R(i|j)$.

In the present works, only electric sectors or electric quadrupole lenses are used. In this case, the conditions $R(x|\gamma) = 0$ and $R(\alpha|\gamma) = 0$ are always fulfilled.

2.2. Ideal perfect space and time focusing condition

In the present article, we are looking for the ideal ion optical systems for a multiturn TOF/MS where ions should return to the point of origin in the system; in other words, the absolute value of the position and angle at the detector plane should be the same of those at initial values in both the horizontal and vertical planes. Such conditions can be expressed using the transfer matrix as

$$R = \begin{pmatrix} \pm 1 & \underline{0} & 0 & 0 & 0 & \underline{0} & 0 \\ \underline{0} & \pm 1 & 0 & 0 & 0 & \underline{0} & 0 \\ 0 & 0 & \pm 1 & \underline{0} & 0 & 0 & 0 \\ 0 & 0 & \underline{0} & \pm 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ \underline{0} & \underline{0} & 0 & 0 & R(l|\gamma) & \underline{0} & 1 \end{pmatrix} \tag{4}$$

It should be noted here that the character $\underline{0}$ (zero with underline) means the matrix element which should be forced to be zero, while 0 (zero without underline) always means zero from the definition. Angular focusing [$R(x|\alpha) = 0$], energy focusing [$R(x|\delta) = 0$], and the condition $R(x|x) = \pm 1$ for lateral magnification are required to conserve the absolute value of the lateral deviation ($|x| = |x_0|$) in the horizontal direction. In the same way, angular focusing [$R(y|\beta) = 0$] and the condition $R(y|y) = \pm 1$ for lateral magnification are required to conserve the absolute value of the lateral deviation ($|y| = |y_0|$) in the vertical direction. Moreover $R(\alpha|x) = R(\alpha|\delta) = 0$ and $R(\alpha|\alpha) = \pm 1$ are required to conserve the absolute value of the angle ($|\alpha| = |\alpha_0|$) in the horizontal direction and $R(\beta|y) = 0$ and

$R(\beta|\beta) = \pm 1$ are required to conserve the absolute value of the angle ($|\beta| = |\beta_0|$) in the vertical direction. In the case of TOF/MS, triple time focusing $R(l|x) = R(l|\alpha) = R(l|\delta) = 0$ is also required. Accordingly, we require the "ninefold focusing," i.e. the nine $\underline{0}$ elements should be zero to satisfy the perfect space and time focusing. The final goal is to find the ion optical systems whose overall transfer matrix becomes as above. Here, we named the system whose magnification is $(+1)$ as normal image type (N type) and the (-1) system as inverse image type (I type).

In our experience, it is easy to find the solution of $R(x|x) = \pm 1$ and $R(x|\alpha) = 0$; however, it is very difficult to satisfy these parameters with $R(\alpha|x) = 0$ simultaneously. In order to overcome this difficulty, symmetrical geometries were introduced. We will explain the detailed process of how to find such solutions.

2.3. The transfer matrices of symmetrical geometry systems

The symmetrical arrangement is indispensable because the multiple focusing can be easily satisfied under few conditions. In this article, we will treat only symmetrical systems consisting of electric fields.

2.3.1. Symmetrical system consisting of two units

Generally, a symmetrical system consists of two basic units (elements). A system consisting of four elements, for example, can be understood as a doubly symmetric system of two units. Therefore, it may be worthwhile to explain more details about the characteristics of a symmetrical arrangement consisting of two units.

Here we define the matrix A as a basic unit which is usually obtained by the multiplication of the matrices of drift spaces, electric sectors, or electric quadrupole lenses. We can derive the point-symmetric matrix A^* of A and the plane-symmetric matrix A^- from the matrix A based on the concept of inverse matrix. They are given in explicit form in the Appendix of [4].

The total transfer matrix of a point symmetric system, A^*A and that of a plane symmetric system, A^-A, can be obtained as

$$A^*A = \begin{pmatrix} A(x|x)A(\alpha|\alpha) + A(x|\alpha)A(\alpha|x) & 2A(x|\alpha)A(\alpha|\alpha) & 0 & 0 & 0 & 2A(x|\delta)A(\alpha|\alpha) & 0 \\ 2A(x|x)A(\alpha|x) & A(x|x)A(\alpha|\alpha) + A(x|\alpha)A(\alpha|x) & 0 & 0 & 0 & 2A(x|\delta)A(\alpha|x) & 0 \\ 0 & 0 & A(y|y)A(\beta|\beta) + A(y|\beta)A(\beta|y) & 2A(y|\beta)A(\beta|\beta) & 0 & 0 & 0 \\ 0 & 0 & 2A(y|y)A(\beta|y) & A(y|y)A(\beta|\beta) + A(y|\beta)A(\beta|y) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ -4A(x|\delta)A(\alpha|x) & -4A(x|\delta)A(\alpha|\alpha) & 0 & 0 & 2A(l|\gamma)2A(l|\delta) - 4A(x|\delta)A(\alpha|\delta) & 1 \end{pmatrix}$$

$$\equiv \begin{pmatrix} [A^*A](x|x) & [A^*A](x|\alpha) & 0 & 0 & [A^*A](x|\gamma) & [A^*A](x|\delta) & 0 \\ [A^*A](\alpha|x) & [A^*A](\alpha|\alpha) & 0 & 0 & [A^*A](\alpha|\gamma) & [A^*A](\alpha|\delta) & 0 \\ 0 & 0 & [A^*A](y|y) & [A^*A](y|\beta) & 0 & 0 & 0 \\ 0 & 0 & [A^*A](\beta|y) & [A^*A](\beta|\beta) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ [A^*A](l|x) & [A^*A](l|\alpha) & 0 & 0 & [A^*A](l|\gamma) & [A^*A](l|\delta) & 1 \end{pmatrix} \tag{5}$$

$$A^-A = \begin{pmatrix} A(x|x)A(\alpha|\alpha) + A(x|\alpha)A(\alpha|x) & 2A(x|\alpha)A(\alpha|\alpha) & 0 & 0 & 0 & 2A(x|\alpha)A(\alpha|\delta) & 0 \\ 2A(x|x)A(\alpha|x) & A(x|x)A(\alpha|\alpha) + A(x|\alpha)A(\alpha|x) & 0 & 0 & 0 & 2A(x|x)A(\alpha|\delta) & 0 \\ 0 & 0 & A(y|y)A(\beta|\beta) + A(y|\beta)A(\beta|\beta) & 2A(y|\beta)A(\beta|\beta) & 0 & 0 & 0 \\ 0 & 0 & 2A(y|y)A(\beta|y) & A(y|y)A(\beta|\beta) + A(y|\beta)A(\beta|\beta) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 4A(x|x)A(\alpha|\delta) & 4A(x|\alpha)A(\alpha|\delta) & 0 & 0 & 2A(l|\gamma)2A(l|\delta) + 4A(x|\delta)A(\alpha|\delta) & 1 \end{pmatrix}$$

$$\equiv \begin{pmatrix} [A^-A](x|x) & [A^-A](x|\alpha) & 0 & 0 & [A^-A](x|\gamma) & [A^-A](x|\delta) & 0 \\ [A^-A](\alpha|x) & [A^-A](\alpha|\alpha) & 0 & 0 & [A^-A](\alpha|\gamma) & [A^-A](\alpha|\delta) & 0 \\ 0 & 0 & [A^-A](y|y) & [A^-A](y|\beta) & 0 & 0 & 0 \\ 0 & 0 & [A^-A](\beta|y) & [A^-A](\beta|\beta) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ [A^-A](l|x) & [A^-A](l|\alpha) & 0 & 0 & [A^-A](l|\gamma) & [A^-A](l|\delta) & 1 \end{pmatrix} \tag{6}$$

where the following relationship [8] between the matrix elements of space and time terms are effectively used to simplify the matrix elements

$$A(l|x) = 2[A(x|x)A(\alpha|\delta) - A(x|\delta)A(\alpha|x)]$$

$$A(l|\alpha) = 2[A(x|\alpha)A(\alpha|\delta) - A(x|\delta)A(\alpha|\alpha)] \quad (7)$$

In addition, the conservation of phase-space volume

$$A(x|x)A(\alpha|\alpha) - A(x|\alpha)A(\alpha|x) = 1$$

$$A(y|y)A(\beta|\beta) - A(y|\beta)A(\beta|y) = 1 \quad (8)$$

are also effectively used.

2.3.2. The perfect space focusing condition

One can easily find from Eqs. (5), (6) that the perfect space focusing conditions in the horizontal plane are expressed as

$$\begin{aligned}
&[A^*A](x|\alpha) = [A^-A](x|\alpha) = 2A(x|\alpha)A(\alpha|\alpha) = 0\\
&[A^*A](\alpha|x) = [A^-A](\alpha|x) = 2A(x|x)A(\alpha|x) = 0\\
&[A^*A](x|\delta) = 2A(x|\delta)A(\alpha|\alpha) = 0\\
&[A^*A](\alpha|\delta) = 2A(x|\delta)A(\alpha|x) = 0\\
&[A^-A](x|\delta) = 2A(x|\alpha)A(\alpha|\delta) = 0\\
&[A^-A](\alpha|\delta) = 2A(x|x)A(\alpha|\delta) = 0
\end{aligned} \quad (9a)$$

In the vertical direction, a similar consideration should be satisfied independently

$$\begin{aligned}
&[A^*A](y|\beta) = [A^-A](y|\beta) = 2A(y|\beta)A(\beta|\beta) = 0\\
&[A^*A](\beta|y) = [A^-A](\beta|y) = 2A(y|y)A(\beta|y) = 0
\end{aligned} \quad (9b)$$

Accordingly, the four cases in Table 1(a) are the only solutions to fulfill the perfect focusing conditions for the symmetric system. One can understand that there are two cases: $A(x|\alpha) = 0$ or $A(\alpha|\alpha) = 0$ to satisfy Eq. (9a1). In the case $A(x|\alpha) = 0$, $A(x|x)$ is not zero; otherwise Eq. (8) is not satisfied. As a consequence, $A(\alpha|\alpha)$ should be zero to satisfy Eq. (9a2). In the same manner, if $A(\alpha|\alpha) = 0$ in Eq. (9a1), then $A(x|x)$ should be zero.

It can be understood that these types of solutions correspond to the focusing status at the intermediate point, namely, whether ion beams are focussed $[A(x|\alpha) = 0]$ or parallel $[A(\alpha|\alpha) = 0]$. We express these differences by using the symbol "●" for focusing and "○" for parallel at the intermediate point. For example, the type of "point symmetry system with intermediate image" can be expressed as $A^* \bullet A$ and the type of "plane symmetry system without intermediate image" where the ion beams are parallel at the intermediate point can be expressed as $A^- \circ A$.

Similar requirements should be satisfied in the vertical direction as shown in Table 1(b). Since there is no energy dispersion in the vertical direction, the required conditions of a point symmetric system are identical to those of a plane symmetric system.

Now, the question becomes how to find the system which satisfies the condition given in Table 1. To our knowledge, it is very difficult to find the case of normal image (N type): $A(x|\alpha) = A(\alpha|x) = 0$. On the other hand, we can easily find the solution of the case of inverse image (I type): $A(x|x) = A(a|a) = 0$ by adding to the drift space before and after an

Table 1a
Four types of solution for the perfect space focusing conditions in horizontal plane

Type	Required conditions	Characteristics	
Point symmetry			
PX1	$A(x\|x) = 0$, $A(\alpha\|\alpha) = 0$, $A(x\|\delta) = 0$	$A^* \bigcirc A$ I type	
PX2	$A(x\|\alpha) = 0$, $A(\alpha\|x) = 0$, $A(x\|\delta) = 0$	$A^* \bullet A$ N type	
Plane symmetry			
LX1	$A(x\|x) = 0$, $A(\alpha\|\alpha) = 0$, $A(\alpha\|\delta) = 0$	$A^- \bigcirc A$ I type	
LX2	$A(x\|\alpha) = 0$, $A(\alpha\|x) = 0$, $A(\alpha\|\delta) = 0$	$A^- \bullet A$ N type	

Table 1b
Two types of solution for the perfect focusing conditions in vertical plane. The symbol s means the symmetries, "*" or "−"

Type	Required conditions	Characteristics
Y1	$A(y\|y) = 0$, $A(\beta\|\beta) = 0$	$A^s \bigcirc A$ I type
Y2	$A(y\|\beta) = 0$, $A(\beta\|y) = 0$	$A^s \bullet A$ N type

arbitrary element and choosing a suitable length for each drift space. These processes can be easily understood in the following transfer matrix multiplication:

$$A = \begin{bmatrix} 1 & L_2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} B(x|x) & B(x|\alpha) & B(x|\delta) \\ B(\alpha|x) & B(\alpha|\alpha) & B(\alpha|\delta) \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & L_1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} B(x|x) + B(\alpha|x)L_2 & B(x|x)L_1 + B(x|\alpha) + [B(\alpha|x)L_1 + B(\alpha|\alpha)]L_2 & B(x|\delta) + B(\alpha|\delta)L_2 \\ B(\alpha|x) & B(\alpha|x)L_1 + B(\alpha|\alpha) & B(\alpha|\delta) \\ 0 & 0 & 1 \end{bmatrix} \quad (10)$$

Here, the matrix B is the transfer matrix of an arbitrary element. Then, by choosing the length of drift spaces as $L_1 = -B(\alpha|\alpha)/B(\alpha|x)$ and $L_2 = -B(x|x)/B(\alpha|x)$, the condition $A(x|x) = A(\alpha|\alpha) = 0$ can be satisfied. In contrast, $A(\alpha|x) = B(\alpha|x)$ is independent of L_1 and L_2. Therefore, it is difficult to satisfy $A(\alpha|x) = 0$.

However, it is not necessary to pursue the solution $A(x|x) = A(\alpha|\alpha) = 0$, following the manner described above. It becomes clear that we can find a reasonable solution for even a system of the fixed drift length by choosing suitable parameters for quadruple lenses or electric sector fields.

2.3.3. The perfect time focusing condition

In an ideal TOF/MS, all ions of the same mass which leave the ion source slit under different initial conditions (position, angle, and energy) arrive at the detector profile plane simultaneously. Such a time focusing condition is expressed from Eqs. (5) for a point symmetric system A^*A as follows:

$$[A^*A](l|x) = -4A(x|\delta)A(\alpha|x) = 0$$

$$[A^*A](l|\alpha) = -4A(x|\delta)A(\alpha|\alpha) = 0$$

$$[A^*A](l|\delta) = 2A(l|\delta) - 4A(x|\delta)A(\alpha|\delta) = 0 \quad (11a)$$

Since the condition $A(\alpha|x) = A(\alpha|\alpha) = 0$ is not permitted from Eq. (8), the requirements for the matrix A of the basic units are $A(x|\delta) = A(l|\delta) = 0$ for a point symmetric system to satisfy the time focusing condition of Eqs. (11a). For a plane symmetric system, the time focusing condition is expressed as follows from Eqs. (6):

$$[A^-A](l|x) = 4A(x|x)A(\alpha|\delta) = 0$$

$$[A^-A](l|\alpha) = 4A(x|\alpha)A(\alpha|\delta) = 0$$

$$[A^-A](l|\delta) = 2A(l|\delta) + 4A(x|\delta)A(\alpha|\delta) = 0 \quad (11b)$$

Similarly, because $A(x|x) = A(x|\alpha) = 0$ is not permitted, the requirements for the matrix A of the basic units are $A(\alpha|\delta) = A(l|\delta) = 0$ for a point symmetric system to satisfy the time focusing condition of Eqs. (11b).

2.3.4. The final form of requirements on matrix A to satisfy the perfect focusing

In the horizontal direction, the final requirements for the matrix A of the basic unit can be summarized in Table 2. In the vertical direction, there is no time term in first order approximation. Therefore the final requirements are the same as in Table 1(b).

2.4. The multiturn condition

In Sec. 2.3.4, perfect focusing conditions are derived. In addition to these, we discuss the geometrical conditions for multiturn systems, namely the criterion necessary to close the ion optical orbit. These conditions can be expressed as follows.

2.4.1. Superposition of detector slit and source slit

The coordinate of the exit position (x_d, y_d) should coincide with that of the entrance position (x_s, y_s). In

Table 2
The perfect focusing conditions of symmetric system consisting of two units in the horizontal direction

Type	Required conditions	Characteristics
Point symmetry		
PX1	$A(x\|x) = 0,\ A(\alpha\|\alpha) = 0,$ $A(x\|\delta) = 0,\ A(l\|\delta) = 0$	$A^* \bigcirc A$ I type
PX2	$A(x\|\alpha) = 0,\ A(\alpha\|x) = 0,$ $A(x\|\delta) = 0,\ A(l\|\delta) = 0$	$A^* \bullet A$ N type
Plane symmetry		
LX1	$A(x\|x) = 0,\ A(\alpha\|\alpha) = 0,$ $A(\alpha\|\delta) = 0,\ A(l\|\delta) = 0$	$A^- \bigcirc A$ I type
LX2	$A(x\|\alpha) = 0,\ A(\alpha\|x) = 0,$ $A(\alpha\|\delta) = 0,\ A(l\|\delta) = 0$	$A^- \bullet A$ N type

principle, the coordinate $(x_d,\ y_d)$ can be simply calculated after giving the physical parameters of individual optical elements, such as drift length, radius, deflection angle of electrode, etc. It may be easier to introduce a more simple constraint depending on the ion optical configuration such as an oval type or a figure-eight type, etc.

2.4.2. Sum of deflection angle rule

In order to connect smoothly to the second turn, the directions of the main optical axis at the entrance and the exit position should coincide with each other. The angle between the direction of the main optical axis at the entrance and exit positions can be expressed as a function of only the deflection angles of sectors. Consequently, the additional condition to close the orbit can be given explicitly as

$$W_1 + W_2 + \ldots + W_m = 2n\pi (n = 0, 1, 2, \ldots) \tag{12}$$

where W_i is the deflection angle of the sector and it should be a negative value when the deflection of the sector is reversed.

2.5. The double symmetric systems consisting of four units

We have not yet found any symmetric systems consisting of two basic units which satisfy the perfect focus up to the present. In the next step, we introduce "doubly symmetric geometry," namely four units are combined in such way that two units are multiplied by two units. The purposes are: (1) to find a perfect focusing geometry consisting of four units, (2) to find a perfect focusing geometry in the vertical direction as well, and (3) to simplify achievement of a closed trajectory. Generally, there are four types of combinations as:

(a) point symmetric system of point symmetric system: $(A^*A)^*(A^*A)$;
(b) plane symmetric system of point symmetric system: $(A^*A)^-(A^*A)$;
(c) point symmetric system of plane symmetric system: $(A^-A)^*(A^-A)$; and
(d) plane symmetric system of plane symmetric system: $(A^-A)^-(A^-A)$.

The geometries of each of these combinations are shown in Fig. 1. In the following sections, we will discuss the perfect focusing conditions of each system.

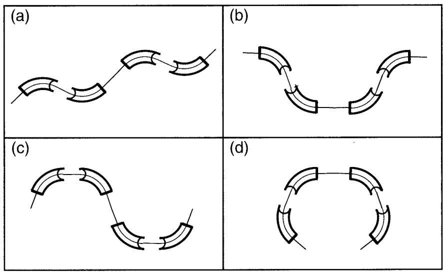

Fig. 1. Four geometrical patterns of symmetric systems consisting of four electric sectors.

Table 3
The perfect focusing conditions of $(A^*A)^*(A^*A)$ geometry in the horizontal direction

Type	Required conditions	Characteristics
AX1	$A(x\|x) = 0,\ A(\alpha\|\alpha) = 0,$ $A(l\|\delta) - 2A(x\|\delta)A(\alpha\|\delta) = 0$	$(A^* \bigcirc A)^* \bullet (A^* \bigcirc A)$ N type
AX2	$A(x\|\alpha) = 0,\ A(\alpha\|x) = 0,$ $A(x\|\delta) = 0,\ A(l\|\delta) = 0$	$(A^* \bullet A)^* \bullet (A^* \bullet A)$ N type
AX3	$2A(x\|x)A(\alpha\|\alpha) - 1 = 0,$ $A(x\|\delta) = 0,\ A(l\|\delta) = 0$	$(A^*A)^* \bigcirc (A^*A)$ I type

2.5.1. Point symmetry of point symmetric system: (A*A)*(A*A)

The overall transfer matrix can be expressed by the elements of the transfer matrix A (first one fourth) as:

$$
\begin{aligned}
&(A^*A)^*(A^*A)\\
&= \begin{bmatrix}
8A(x|x)A(x|\alpha)A(\alpha|x)A(\alpha|\alpha)+1 & 4A(x|\alpha)A(\alpha|\alpha)(2A(x|x)A(\alpha|\alpha)-1) & 0 & 0 & 0 & 4A(x|\delta)A(\alpha|\alpha)(2A(x|x)A(\alpha|\alpha)-1) & 0\\
4A(x|x)A(\alpha|x)(2A(x|x)A(\alpha|\alpha)-1) & 8A(x|x)A(x|\alpha)A(\alpha|x)A(\alpha|\alpha)+1 & 0 & 0 & 0 & 8A(x|x)A(x|\delta)A(\alpha|x)A(\alpha|\alpha) & 0\\
0 & 0 & 8A(y|y)A(y|\beta)A(\beta|y)A(\beta|\beta)+1 & 4A(y|\beta)A(\beta|\beta)(2A(y|y)A(\beta|\beta)-1) & 0 & 0 & 0\\
0 & 0 & 4A(y|y)A(\beta|y)(2A(y|y)A(\beta|\beta)-1) & 8A(y|y)A(y|\beta)A(\beta|y)A(\beta|\beta)+1 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 1 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 1 & 0\\
-16A(x|x)A(x|\delta)A(\alpha|x)A(\alpha|\alpha) & -8A(x|\delta)A(\alpha|\alpha)(2A(x|x)A(\alpha|\alpha)-1) & 0 & 0 & 4A(l|\gamma) & 4A(l|\delta)-8A(x|\delta)(A(\alpha|\delta)+2A(x|\delta)A(\alpha|x)A(\alpha|\alpha)) & 1
\end{bmatrix}
\end{aligned}
\tag{13}
$$

The perfect focusing conditions are

$$
\begin{aligned}
&[(A^*A)^*(A^*A)](x|x) = 8A(x|x)A(x|\alpha)A(\alpha|x)A(\alpha|\alpha) + 1 = \pm 1\\
&[(A^*A)^*(A^*A)](x|\alpha) = 4A(x|\alpha)A(\alpha|\alpha)(2A(x|x)A(\alpha|\alpha) - 1) = 0\\
&[(A^*A)^*(A^*A)](x|\delta) = 4A(x|\delta)A(\alpha|\alpha)(2A(x|x)A(\alpha|\alpha) - 1) = 0\\
&[(A^*A)^*(A^*A)](\alpha|x) = 4A(x|x)A(\alpha|x)(2A(x|x)A(\alpha|\alpha) - 1) = 0\\
&[(A^*A)^*(A^*A)](\alpha|\alpha) = 8A(x|x)A(x|\alpha)A(\alpha|x)A(\alpha|\alpha) + 1 = \pm 1\\
&[(A^*A)^*(A^*A)](\alpha|\delta) = 8A(x|x)A(x|\delta)A(\alpha|x)A(\alpha|\alpha) = 0\\
&[(A^*A)^*(A^*A)](y|y) = 8A(y|y)A(y|\beta)A(\beta|y)A(\beta|\beta) + 1 = \pm 1\\
&[(A^*A)^*(A^*A)](y|\beta) = 4A(y|\beta)A(\beta|\beta)(2A(y|y)A(\beta|\beta) - 1) = 0\\
&[(A^*A)^*(A^*A)](\beta|y) = 4A(y|y)A(\beta|y)(2A(y|y)A(\beta|\beta) - 1) = 0\\
&[(A^*A)^*(A^*A)](\beta|\beta) = 8A(y|y)A(y|\beta)A(\beta|y)A(\beta|\beta) + 1 = \pm 1\\
&[(A^*A)^*(A^*A)](l|x) = -16A(x|x)A(x|\delta)A(\alpha|x)A(\alpha|\alpha) = 0\\
&[(A^*A)^*(A^*A)](l|\alpha) = -8A(x|\delta)A(\alpha|\alpha)(2A(x|x)A(\alpha|\alpha) - 1) = 0\\
&[(A^*A)^*(A^*A)](l|\delta) = 4A(l|\delta) - 8A(x|\delta)(A(\alpha|\delta) + 2A(x|\delta)A(\alpha|x)A(\alpha|\alpha)) = 0
\end{aligned}
\tag{14}
$$

The final required conditions in the horizontal direction are classified in three cases expressed in Table 3. The type of AX1 is similar to the system connecting the type of PX1 in Table 2 with the point symmetric system of PX1. However the number of required conditions to satisfy perfect focusing can be reduced from four to three. The type of AX2 is nothing but the system connecting the type of PX2 with the point symmetric system of PX2. The type of AX3 only appears in the doubly symmetric geometry. Ion beams are parallel at the point after two basic units, while they are not parallel nor focussed after the first units. The number of required conditions can be reduced from four to three.

In the same manner, the final required conditions in the vertical direction are classified in three cases expressed in Table 4. In the vertical direction, as the symmetric system consisting of two units, there is no difference between the point or plane symmetry. Therefore, the same conditions are also required in the following three symmetric systems.

2.5.2. Plane symmetry of point symmetric system: $(A^*A)^-(A^*A)$

The overall transfer matrix of this arrangement is given in the following:

Table 4
The perfect focusing conditions of the symmetric system consisting of four units in the vertical direction. The symbol s means the symmetries "*" or "−"

Type	Required conditions	Characteristics
QY1	$A(y\|y) = 0,\ A(\beta\|\beta) = 0$	$(A^s \bigcirc A)^s \bullet (A^s \bigcirc A)$ N type
QY2	$A(y\|\beta) = 0,\ A(\beta\|y) = 0$	$(A^s \bullet A)^s \bullet (A^s \bullet A)$ N type
QY3	$2A(y\|y)A(\beta\|\beta) - 1 = 0$	$(A^sA)^s \bigcirc (A^sA)$ I type

$$(A^*A)^-(A^*A) =$$

$$\begin{bmatrix} 8A(x|x)A(x|\alpha)A(\alpha|x)A(\alpha|\alpha)+1 & 4A(x|\alpha)A(\alpha|\alpha)(2A(x|x)A(\alpha|\alpha)-1) & 0 & 0 & 0 & 8A(x|\alpha)A(x|\delta)A(\alpha|x)A(\alpha|\alpha) & 0 \\ 4A(x|x)A(\alpha|x)(2A(x|x)A(\alpha|\alpha)-1) & 8A(x|x)A(x|\alpha)A(\alpha|x)A(\alpha|\alpha)+1 & 0 & 0 & 0 & 4A(x|\delta)A(\alpha|x)(2A(x|x)A(\alpha|\alpha)-1) & 0 \\ 0 & 0 & 8A(y|y)A(y|\beta)A(\beta|y)A(\beta|\beta)+1 & 4A(y|\beta)A(\beta|\beta)(2A(y|y)A(\beta|\beta)-1) & 0 & 0 & 0 \\ 0 & 0 & 4A(y|y)A(\beta|y)(2A(y|y)A(\beta|\beta)-1) & 8A(y|y)A(y|\beta)A(\beta|y)A(\beta|\beta)+1) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 8A(x|\delta)A(\alpha|x)(2A(x|x)A(\alpha|\alpha)-1) & 16A(x|\alpha)A(x|\delta)A(\alpha|x)A(\alpha|\alpha) & 0 & 0 & 4A(l|\gamma) & 4A(l|\delta)-8A(x|\delta)(A(\alpha|\delta)-2A(x|\delta)A(\alpha|x)A(\alpha|\alpha)) & 1 \end{bmatrix} \tag{15}$$

The perfect focusing conditions are

$$\begin{aligned}
&[(A^*A)^-(A^*A)](x|x) = 8A(x|x)A(x|\alpha)A(\alpha|x)A(\alpha|\alpha) + 1 = \pm 1 \\
&[(A^*A)^-(A^*A)](x|\alpha) = 4A(x|\alpha)A(\alpha|\alpha)(2A(x|x)A(\alpha|\alpha) - 1) = 0 \\
&[(A^*A)^-(A^*A)](x|\delta) = 8A(x|\alpha)A(x|\delta)A(\alpha|x)A(\alpha|\alpha) = 0 \\
&[(A^*A)^-(A^*A)](\alpha|x) = 4A(x|\delta)A(\alpha|x)(2A(x|x)A(\alpha|\alpha) - 1) = 0 \\
&[(A^*A)^-(A^*A)](\alpha|\alpha) = 8A(x|x)A(x|\alpha)A(\alpha|x)A(\alpha|\alpha) + 1 = \pm 1 \\
&[(A^*A)^-(A^*A)](\alpha|\delta) = 4A(x|\delta)A(\alpha|x)(2A(x|x)A(\alpha|\alpha) - 1) = 0 \\
&[(A^*A)^-(A^*A)](y|y) = 8A(y|y)A(y|\beta)A(\beta|y)A(\beta|\beta) + 1 = \pm 1 \\
&[(A^*A)^-(A^*A)](y|\beta) = 4A(y|\beta)A(\beta|\beta)(2A(y|y)A(\beta|\beta) - 1) = 0 \\
&[(A^*A)^-(A^*A)](\beta|y) = 4A(y|y)A(\beta|y)(2A(y|y)A(\beta|\beta) - 1) = 0 \\
&[(A^*A)^-(A^*A)](\beta|\beta) = 8A(y|y)A(y|\beta)A(\beta|y)A(\beta|\beta) + 1 = \pm 1 \\
&[(A^*A)^-(A^*A)](l|x) = 8A(x|\delta)A(\alpha|x)(2A(x|x)A(\alpha|\alpha) - 1) = 0 \\
&[(A^*A)^-(A^*A)](l|\alpha) = 16A(x|\delta)A(\alpha|\alpha)(A(x|x)A(\alpha|\alpha) - 1) = 0 \\
&[(A^*A)^-(A^*A)](l|\delta) = 4A(l|\delta) - 8A(x|\delta)(A(\alpha|\delta) - 2A(x|\delta)A(\alpha|x)A(\alpha|\alpha)) = 0
\end{aligned} \tag{16}$$

The final required conditions in horizontal directions are classified in three cases expressed in Table 5. The type of BX1 is nothing but the system connecting the type of PX1 in Table 2 with the point symmetric

Table 5
The perfect focusing conditions of $(A^*A)^-(A^*A)$ geometry in the horizontal direction

Type	Required conditions	Characteristics
BX1	$A(x\|x) = 0,\ A(\alpha\|\alpha) = 0,$ $A(x\|\delta) = 0,\ A(l\|\delta) = 0$	$(A^* \bigcirc A)^- \bullet (A^* \bigcirc A)$ N type
BX2	$A(x\|\alpha) = 0,\ A(\alpha\|x) = 0,$ $A(l\|\delta) - 2A(x\|\delta)A(\alpha\|\delta) = 0$	$(A^* \bullet A)^- \bullet (A^* \bullet A)$ N type
BX3	$2A(x\|x)A(\alpha\|\alpha) - 1 = 0,$ $A(x\|\delta) = 0,\ A(l\|\delta) = 0$	$(A^*A)^- \bigcirc (A^*A)$ I type

system of PX1. The type of BX2 is similar to the system connecting the type of PX2 with the plane symmetric system of PX2. However, the number of required conditions to satisfy perfect focusing can be reduced from four to three. The type of BX3 only appears in the doubly symmetric geometry. Ion beams are parallel at the point after two basic units. The number of required conditions can be reduced from four to three.

The final required conditions in the vertical directions are also classified in three cases expressed in Table 4.

2.5.3. Point symmetry of plane symmetric system: $(A^-A)^*(A^-A)$

The overall transfer matrix of this arrangement is given in the following:

$$(A^-A)^*(A^-A) = \begin{bmatrix} 8A(x|x)A(x|\alpha)A(\alpha|x)A(\alpha|\alpha)+1 & 4A(x|\alpha)A(\alpha|\alpha)(2A(x|x)A(\alpha|\alpha)-1) & 0 & 0 & 0 & 4A(x|\alpha)A(\alpha|\delta)(2A(x|x)A(\alpha|\alpha)-1) & 0 \\ 4A(x|x)A(\alpha|x)(2A(x|x)A(\alpha|\alpha)-1) & 8A(x|x)A(x|\alpha)A(\alpha|x)A(\alpha|\alpha)+1 & 0 & 0 & 0 & 8A(x|x)A(x|\alpha)A(\alpha|x)A(\alpha|\delta) & 0 \\ 0 & 0 & 8A(y|y)A(y|\beta)A(\beta|y)A(\beta|\beta)+1 & 4A(y|\beta)A(\beta|\beta)(2A(y|y)A(\beta|\beta)-1) & 0 & 0 & 0 \\ 0 & 0 & 4A(y|y)A(\beta|y)(2A(y|y)A(\beta|\beta)-1) & 8A(y|y)A(y|\beta)A(\beta|y)A(\beta|\beta)+1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ -16A(x|x)A(x|\alpha)A(\alpha|x)A(\alpha|\delta) & -8A(x|\alpha)A(\alpha|\delta)(2A(x|x)A(\alpha|\alpha)-1) & 0 & 0 & 4A(l|\gamma) & 4A(l|\delta)+8A(\alpha|\delta)(A(x|\delta)-2A(x|x)A(x|\alpha)A(\alpha|\delta)) & 1 \end{bmatrix} \tag{17}$$

The perfect focusing conditions are

$$\begin{aligned}
&[(A^*A)^*(A^*A)](x|x) = 8A(x|x)A(x|\alpha)A(\alpha|x)A(\alpha|\alpha) + 1 = \pm 1 \\
&[(A^*A)^*(A^*A)](x|\alpha) = 4A(x|\alpha)A(\alpha|\alpha)(2A(x|x)A(\alpha|\alpha) - 1) = 0 \\
&[(A^*A)^*(A^*A)](x|\delta) = 4A(x|\alpha)A(\alpha|\delta)(2A(x|x)A(\alpha|\alpha) - 1) = 0 \\
&[(A^*A)^*(A^*A)](\alpha|x) = 4A(x|x)A(\alpha|x)(2A(x|x)A(\alpha|\alpha) - 1) = 0 \\
&[(A^*A)^*(A^*A)](\alpha|\alpha) = 8A(x|x)A(x|\alpha)A(\alpha|x)A(\alpha|\alpha) + 1 = \pm 1 \\
&[(A^*A)^*(A^*A)](\alpha|\delta) = 8A(x|x)A(x|\alpha)A(\alpha|x)A(\alpha|\delta) = 0 \\
&[(A^*A)^*(A^*A)](y|y) = 8A(y|y)A(y|\beta)A(\beta|y)A(\beta|\beta) + 1 = \pm 1 \\
&[(A^*A)^*(A^*A)](y|\beta) = 4A(y|\beta)A(\beta|\beta)(2A(y|y)A(\beta|\beta) - 1) = 0 \\
&[(A^*A)^*(A^*A)](\beta|y) = 4A(y|y)A(\beta|y)(2A(y|y)A(\beta|\beta) - 1) = 0 \\
&[(A^*A)^*(A^*A)](\beta|\beta) = 8A(y|y)A(y|\beta)A(\beta|y)A(\beta|\beta) + 1 = \pm 1 \\
&[(A^*A)^*(A^*A)](l|x) = -16A(x|x)A(x|\alpha)A(\alpha|x)A(\alpha|\delta) = 0 \\
&[(A^*A)^*(A^*A)](l|\alpha) = -8A(x|\alpha)A(\alpha|\delta)(2A(x|x)A(\alpha|\alpha) - 1) = 0 \\
&[(A^*A)^*(A^*A)](l|\delta) = 4A(l|\delta) + 8A(\alpha|\delta)(A(x|\delta) - 2A(x|x)A(x|\alpha)A(\alpha|\delta)) = 0
\end{aligned} \tag{18}$$

Table 6
The perfect focusing conditions of $(A^{-}A)^{*}(A^{-}A)$ geometry in the horizontal direction

Type	Required conditions	Characteristics
CX1	$A(x\|x) = 0,\ A(\alpha\|\alpha) = 0,$ $A(\alpha\|\delta) = 0,\ A(l\|\delta) = 0$	$(A^{-} \bigcirc A)^{*} \bullet (A^{-} \bigcirc A)$ N type
CX2	$A(x\|\alpha) = 0,\ A(\alpha\|x) = 0,$ $A(l\|\delta) + 2A(x\|\delta)A(\alpha\|\delta) = 0$	$(A^{-} \bullet A)^{*} \bullet (A^{-} \bullet A)$ N type
CX3	$2A(x\|x)A(\alpha\|\alpha) - 1 = 0,$ $A(\alpha\|\delta) = 0,\ A(l\|\delta) = 0$	$(A^{-}A)^{*} \bigcirc (A^{-}A)$ I type

The final required conditions in the horizontal directions are classified in three cases expressed in Table 6. The type of CX1 is nothing but the system connecting the type of LX1 in Table 2 with the plane symmetric system of LX1. The type of CX2 is similar to the system connecting the type of LX2 with the plane symmetric system of LX2. However, the number of required conditions to satisfy perfect focusing can be reduced from four to three. The type of CX3 only appears in the doubly symmetric geometry. Ion beams are parallel at the point after two basic units. The number of required conditions can be reduced from four to three.

The final required conditions in the vertical directions are also classified in three cases expressed in Table 4.

2.5.4. Plane symmetry of plane symmetric system: $(A^{-}A)^{-}(A^{-}A)$

The overall transfer matrix of this arrangement is given in the following:

$$(A^{-}A)^{-}(A^{-}A)$$
$$= \begin{bmatrix} 8A(x|x)A(x|\alpha)A(\alpha|x)A(\alpha|\alpha)+1 & 4A(x|\alpha)A(\alpha|\alpha)(2A(x|x)A(\alpha|\alpha)-1) & 0 & 0 & 0 & 8A(x|x)A(x|\alpha)A(\alpha|\alpha)A(\alpha|\delta) & 0 \\ 4A(x|x)A(\alpha|x)(2A(x|x)A(\alpha|\alpha)-1) & 8A(x|x)A(x|\alpha)A(\alpha|x)A(\alpha|\alpha)+1 & 0 & 0 & 0 & 4A(x|x)A(\alpha|\delta)(2A(x|x)A(\alpha|\alpha)-1) & 0 \\ 0 & 0 & 8A(y|y)A(y|\beta)A(\beta|y)A(\beta|\beta)+1 & 4A(y|\beta)A(\beta|\beta)(2A(y|y)A(\beta|\beta)-1) & 0 & 0 & 0 \\ 0 & 0 & 4A(y|y)A(\beta|y)(2A(y|y)A(\beta|\beta)-1) & 8A(y|y)A(y|\beta)A(\beta|y)A(\beta|\beta)+1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 8A(x|x)A(\alpha|\delta)(2A(x|x)A(\alpha|\alpha)-1) & 16A(x|x)A(x|\alpha)A(\alpha|\alpha)A(\alpha|\delta) & 0 & 0 & 4A(l|\gamma) & 4A(l|\delta)+8A(\alpha|\delta)(A(x|\delta)+2A(x|x)A(x|\alpha)A(\alpha|\delta)) & 1 \end{bmatrix} \tag{19}$$

The perfect focusing conditions are

$$\begin{aligned}
[(A^{*}A)^{-}(A^{*}A)](x|x) &= 8A(x|x)A(x|\alpha)A(\alpha|x)A(\alpha|\alpha) + 1 = \pm 1 \\
[(A^{*}A)^{-}(A^{*}A)](x|\alpha) &= 4A(x|\alpha)A(\alpha|\alpha)(2A(x|x)A(\alpha|\alpha) - 1) = 0 \\
[(A^{*}A)^{-}(A^{*}A)](x|\delta) &= 8A(x|x)A(x|\alpha)A(\alpha|\alpha)A(\alpha|\delta) = 0 \\
[(A^{*}A)^{-}(A^{*}A)](\alpha|x) &= 4A(x|\delta)A(\alpha|x)(2A(x|x)A(\alpha|\alpha) - 1) = 0 \\
[(A^{*}A)^{-}(A^{*}A)](\alpha|\alpha) &= 8A(x|x)A(x|\alpha)A(\alpha|x)A(\alpha|\alpha) + 1 = \pm 1 \\
[(A^{*}A)^{-}(A^{*}A)](\alpha|\delta) &= 4A(x|x)A(\alpha|\delta)(2A(x|x)A(\alpha|\alpha) - 1) = 0 \\
[(A^{*}A)^{-}(A^{*}A)](y|y) &= 8A(y|y)A(y|\beta)A(\beta|y)A(\beta|\beta) + 1 = \pm 1 \\
[(A^{*}A)^{-}(A^{*}A)](y|\beta) &= 4A(y|\beta)A(\beta|\beta)(2A(y|y)A(\beta|\beta) - 1) = 0 \\
[(A^{*}A)^{-}(A^{*}A)](\beta|y) &= 4A(y|y)A(\beta|y)(2A(y|y)A(\beta|\beta) - 1) = 0 \\
[(A^{*}A)^{-}(A^{*}A)](\beta|\beta) &= 8A(y|y)A(y|\beta)A(\beta|y)A(\beta|\beta) + 1 = \pm 1 \\
[(A^{*}A)^{-}(A^{*}A)](l|x) &= 8A(x|x)A(\alpha|\delta)(2A(x|x)A(\alpha|\alpha) - 1) = 0 \\
[(A^{*}A)^{-}(A^{*}A)](l|\alpha) &= 16A(x|x)A(x|\alpha)A(\alpha|\alpha)A(\alpha|\delta) = 0 \\
[(A^{*}A)^{-}(A^{*}A)](l|\delta) &= 4A(l|\delta) + 8A(\alpha|\delta)(A(x|\delta) + 2A(x|x)A(x|\alpha)A(\alpha|\delta)) = 0
\end{aligned} \tag{20}$$

Table 7
The perfect focusing conditions of $(A^{-}A)^{-}(A^{-}A)$ geometry in the horizontal direction

Type	Required conditions	Characteristics
DX1	$A(x\|x) = 0$, $A(\alpha\|\alpha) = 0$, $A(l\|\delta) + 2A(x\|\delta)A(\alpha\|\delta) = 0$	$(A^{-} \bigcirc A)^{-} \bullet (A^{-} \bigcirc A)$ N type
DX2	$A(x\|\alpha) = 0$, $A(\alpha\|x) = 0$, $A(\alpha\|\delta) = 0$, $A(l\|\delta) = 0$	$(A^{-} \bullet A)^{-} \bullet (A^{-} \bullet A)$ N type
DX3	$2A(x\|x)A(\alpha\|\alpha) - 1 = 0$, $A(\alpha\|\delta) = 0$, $A(l\|\delta) = 0$	$(A^{-}A)^{-} \bigcirc (A^{-}A)$ I type

The final required conditions in horizontal directions are classified in three cases expressed in Table 7. The type of DX1 is similar to the system connecting the type of LX1 in Table 2 with the plane symmetric system of LX1. However, the number of required conditions to satisfy perfect focusing can be reduced from four to three. The type of DX2 is nothing but the system connecting the type of LX2 with the plane symmetric system of LX2. The type of DX3 only appears in the doubly symmetric geometry. Ion beams are parallel at the point after two basic units. The number of required conditions can be reduced from four to three.

The final required conditions in the vertical directions are also classified in three cases expressed in Table 4.

3. Conclusion

We have investigated the required conditions for the perfect space and time focusing for a multiturn TOF mass spectrometer. In this study we treated symmetrical systems consisting of two and four basic units. It was shown that the multiple focusing can be satisfied under few conditions by introducing symmetry in the arrangement. In the case of the system consisting of four basic units, the required conditions can be reduced compared with the case of the system consisting of only two basic units.

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